Journal of Experimental Psychology:
Human Perception and Performance
1980, Vol. 6, No. 2, 330–354

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Comparing Parallel and Serial Models: Theory and Implementation

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The present article addresses the serial/parallel processing question at both a theoretical and an empirical level. First, we review some general distribution-free properties of parallel and serial models. Next, we derive predictions for mean reaction times (RTs) for parallel models for both unlimited and limited capacity conditions. We show that when processing times differ across same and different comparisons and across spatial locations, serial and parallel models are identifiable at the level of mean RTs. Data from two experiments, which include representative samples of such widely used tasks as memory and visual search and same–different comparisons, clearly ruled out exhaustive models in favor of self-terminating models. A self-terminating serial model fits mean RTs better than a fixed limited capacity parallel model across both experiments and across two levels of complexity of the two models.

The present article has two purposes: first, to review general properties of parallel and serial models with particular attention to the capacity issue for parallel models; and second, to develop and test explicit classes of serial and parallel models whose qualitative features capture the pattern of reaction times (RTs) across conditions of a comprehensive paradigm.

The capacity issue is a crucial one for comparing serial and parallel models. Though a serial model is, by definition, a limited capacity model (since attention or processing effort is devoted to one item at a time), parallel models with limitations on capacity can produce mean RT predictions that are indistinguishable from those for serial models. Although these results have caused some investigators to despair of ever distinguishing parallel from serial models, they can be distinguished, as we show below, if same and different comparison rates differ and serial position effects are observed. Serial position effects can be obtained when there is a preferred order of processing in a serial model or when there is a nonuniform distribution of attention across the potential set of comparisons in a parallel model.

Previously, detailed predictions for parallel limited capacity models have not been presented, except in general terms (e.g., Townsend, 1974). The reasons for this are not hard to find. First, the equations quickly become very complicated for all but the simplest conditions, and second, it is difficult to decide exactly how the number of potential comparisons might limit capacity. In the present article, we make the fol-

This research was supported in part by National Institute for Mental Health Research Grant MH-20723 to the first author, and in part by National Science Foundation Grant 76-84053 to the second author. A preliminary report of this research was presented at an informal symposium at the meeting of the American Psychological Association, New York, August 1974, and at a Symposium of the Psycholinguistics Circle of New York at New York University, October 1973.

We wish to thank Dirk Vorberg for valuable advice on both the theoretical and practical level, Jean-Claude Falmagne and Murray Glanzer for critical comments on an earlier version of this article, and Micha Razel for help with computer programming.

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lowing simplifying assumptions in deriving parallel predictions for latency data: (a) that intercompletion times are exponentially distributed, which makes the parallel prediction equations expressible in closed form, and (b) that capacity is divided by the number of potential comparisons. Assumption b leads to what we call a fixed limited capacity parallel model.

To test various classes of the serial and parallel models, we collected RT data within a paradigm including instances of short-term memory search, visual search, and simple, conjunctive, and disjunctive same–different judgments. The general pattern of RTs across conditions rejected all exhaustive models (both parallel and serial) in favor of self-terminating models. Strong and consistent serial position effects within given conditions permitted us to distinguish serial from limited capacity parallel self-terminating models at a finer level of analysis. Quantitative comparisons between the two classes of models showed that a serial self-terminating model fit the data better than a fixed limited capacity parallel model.

Although the results of statistical tests favor the serial model, we feel that this is neither the most important result of our reserch nor the last word on the parallel/serial issue. Rather, we feel that the important contributions of this article are heuristic in illustrating how parallel models might be developed, what aspects of data might be used to narrow the field of potential models, and what assumptions need to be added to completely characterize a particular processing strategy.

The article is organized in the following manner. First, we briefly review the empirical literature on the serial/parallel issue; next, we review general properties of serial and parallel models; finally, we describe the design and results of the experimental tests of the models.

Literature Review

An impressive array of experimental paradigms employing reaction time have been analyzed to find out whether subjects are employing parallel or serial comparison strategies and whether the task is accomplished with a self-terminating or exhaustive criterion. These include simple same–different judgment tasks for pairs of multi-dimensional stimuli presented simultaneously or successively (e.g., Bamber, 1969; Egeth, 1966; Snodgrass, 1972a), visual search for one or several targets (e.g., Neisser, 1963a; Neisser, Novick, & Lazar, 1963; Atkinson, Holmgren, & Juola, 1969; Townsend & Roos, 1973), short- and long-term memory search (Atkinson & Juola, 1974; Sternberg, 1966, 1975), and conjunctive and disjunctive same–different judgment tasks (e.g., Briggs & Blaha, 1969; Marcel, 1970; Nickerson, 1967; Snodgrass, 1972b; Taylor, 1976a).

Almost all of these tasks involve presenting some number, N, of stimuli to be stored in memory and then presenting some number, M, of stimuli in a visual display and asking the subject whether $1 \ldots m$ of the stimuli in the visual display "match" (usually, are identical to) $1 \ldots n$ of the stimuli in memory. Elsewhere (Snodgrass, 1972b) these tasks have been designated memory $N{:}M$ tasks, in which N items are in memory and M items are in the visual display.

In this terminology, same–different tasks for successively presented items are denoted 1:1 because one item is in short-term memory and the second is in a visual display; visual scanning experiments are denoted $1{:}M$ if a single item is being searched for and $N{:}M$ if more than a single item is the object of search; and memory scanning experiments are denoted $N{:}1$ because N items are in short- or long-term memory and a single item is in a visual display.

General Properties of Serial and Parallel Models

Several issues that are relevant to the serial/parallel issue will be taken up before examining the characteristics of various models of the comparison process. Townsend has formalized some of the mathematics and reasoning necessary for the study of parallel and serial processes (see, e.g., Townsend, 1971, 1972, 1974, 1976a, 1976b).

We treat in some detail various aspects of the serial/parallel issue to set the stage for what is to come. A general discussion of the possible classes of serial and parallel models is important in clarifying how predictions were derived for the rather complex set of conditions that were used.

Units of Analysis

Critial to specification of the aspects of processing in any cognitive setting is the presumed unit of analysis (e.g., Taylor, 1976b). It should be apparent that at some level of analysis, items are dealt with in parallel. That is, a line or curve is probably analyzed in parallel, rather than being decomposed into some finer units such as points in a Cartesian space. At the other extreme, there are clearly cases in which items are decomposed into components and are dealt with in a serial fashion. For example, visually presented sentences are surely not dealt with in a completely parallel fashion. Between these two extremes, however, there might be cases in which the type of processing may depend on the nature of the material, the degree of learning, and the nature of the task (e.g., Schneider & Shiffrin, 1977).

Obviously, the choice of the unit of analysis will affect decisions about whether the task is done in serial or in parallel. In simple same–different tasks in which single items are being compared, the unit of analysis typically is taken to be the features of the single items—either features of single letters or digits, or dimensions of multidimensional visual forms. In contrast, in visual scanning or memory scanning experiments, the unit of analysis is usually the entire single item (digits, letters, patterns, etc.).

Serial Versus Parallel Comparisons

The question of whether the mind can deal with more than one thing at a time has a very long history and was experimentally studied via the span of apprehension. Implicit in some early philosophizing about the issue was the concept that although the mind could deal with several items at one time, there was a spread of attention across them. This type of system, in which items are processed in parallel but with a concomitant decrement in the degree of attention (or clarity) that each can receive as the number to be processed increases, is known as a parallel limited capacity system. The question of capacity is so closely linked with the parallel–serial issue that both will be considered together.

More attention has been given in the literature to serial than to parallel models for various kinds of tasks, primarily because the prediction equations for parallel models are difficult except in a few simple cases. Two issues for parallel models are important: One is the limited versus unlimited capacity issue referred to above, and the second is the form of the distribution of the comparison times.

One simple model is that in which capacity is unlimited and the comparison times are constant and identical (deterministic) for all items to be processed. In a situation in which all comparisons must be completed before a decision can be made, such a model predicts that RT will remain constant as the number of items in a visual or memory display increase. Evidence for such a model has been reported for a visual search task by Egeth, Jonides, and Wall (1972). Yet, stochastic (nondeterministic) models can easily be found that also predict such flat mean RT functions (e.g., Townsend, 1974, p. 162).

When the comparison times are distributed exponentially, the comparisons are independent, and the capacity is unlimited, the time for exhaustive scanning of n items increases approximately with $\log n$ (Christie & Luce, 1956; Rapoport, 1959); and when no particular distribution is assumed for the comparison times but the independence assumption is retained, an upper bound on the maximum increase in RT can be determined (Gumbel, 1954).

Self-Terminating Versus Exhaustive Comparisons

A self-terminating system is one in which the search is terminated whenever it is logically possible, whereas an exhaustive system is one in which all items are compared regardless of the logical possibility of stopping prior to all n comparisons. However, even a fundamentally self-terminating search may logically need to be exhaustive, in the sense that all n items need to be processed. For example, if subjects are assumed to store only positive set items in memory in a memory-scanning task, then for a self-terminating system the search is self-terminating on positive trials but exhaustive (in the sense of requiring search through all items) on negative trials. It is possible, however, to build models for item sets consisting of only a few items (such as the digits 0–9) in which both positive and negative items are stored in memory, and subjects self-terminate on both positive and negative trials (Theios, Smith, Haviland, Traupmann, & Moy, 1973). Here we will refer to self-terminating systems as those that terminate as soon as it is logically possible, whether that terminating point is after all n items are searched or only a subset of the n items is searched, whereas exhaustive systems are those in which all n items are always searched.

Logical Stopping Rules

In the following sections, we describe simplified predictions for three logical stopping rules for the following models: serial self-terminating, serial exhaustive, parallel self-terminating unlimited capacity, parallel exhaustive unlimited capacity, parallel self-terminating limited capacity, and parallel exhaustive limited capacity. We do not make the limited–unlimited capacity distinction for serial models. Plausible serial models are, almost by definition, limited capacity systems, since they imply that subjects can only deal with items one at a time.

We consider predictions for the following three stopping rules: (a) All of n must finish, a stopping rule that is appropriate for certain situations in self-terminating systems and is appropriate for all situations for exhaustive systems. (b) One particular element of n must finish, which is appropriate for self-terminating systems on positive trials. (c) Any one of n must finish. Stopping Rule c is one that is not logically required in many experimental paradigms, but it is appropriate for one we shall consider in detail later. One example of such a stopping rule can be found in a task used by Bamber (1969) in which subjects were presented with two strings of four letters that could be completely identical or could differ in one, two, three, or all four letters. The case in which all four are different embodies the situation in which, in a self-terminating system, the subject could stop when any one of the n comparisons finishes. A stopping rule intermediate between b and c is any m of n must finish, where $m < n$. For simplicity we do not consider this intermediate rule.

In addition, we consider predictions from the models only for mean latencies. A number of other aspects of the latency distributions, such as the minimum and maximum times, have been shown to be of importance in distinguishing self-terminating from exhaustive models (e.g., Sternberg, 1975); however, we do not here consider those aspects of the RT distributions. In addition, it may be possible eventually to distinguish models on the basis of combined RT and error rate information. However, most of the situations that we consider attempt to keep error rates to a minimum, so we will choose to ignore errors in the following analyses. Although certain useful generalized remarks may be made concerning speed–accuracy trade-offs (e.g., Pachella, 1974), no detailed analyses on both RTs and error rates can be performed in the absence of a well-specified quantitative model. We view the latency characteristics of the present models as important in their own right and also as propaedeutic to development of a complete theory embracing both types of information.

We note that the latency predictions for the parallel models are based on the assumption that comparison times are exponentially distributed, but the serial predictions are distribution-free. It is possible to generalize parallel results based on exponential distributions to other kinds of comparison time distributions (e.g., Townsend,

Table 1
Reaction Time Predictions for Serial and Parallel Models for Three Stopping Rules

	Model			
		Parallel		
Stopping rule	Serial	Unlimited capacity	Limited fixed capacity	Limited reallocatable capacity
All of n must finish (exhaustive)	nT	$(\log n)\ T$	$(n \log n)\ T$	nT
One particular of n must finish (self-terminating)	$[(n+1)/2]\ T$	T	nT	$[(n+1)/2]\ T$
Any one of n must finish	T	$(1/n)\ T$	T	T

Note. T = mean time for one comparison; n = number of comparisons.

1976a; Townsend & Ashby, 1978). The exponential models often yield behavior not atypical of parallel models in general, and they have the advantage of being mathematically tractable.[1]

Table 1 presents latency predictions for serial self-terminating and unlimited and limited capacity parallel self-terminating models for the three stopping rules. Predictions for the corresponding exhaustive models are always those for Stopping Rule a, all of n must finish. The predictions are expressed in terms of T, the mean time for a single comparison, and n, the number of comparisons being made. It may be noted that for simplicity, the processing rates are assumed equal across the item positions. The behavior of the mean RT as a function of n (e.g., increasing vs. decreasing; positively vs. negatively accelerated; etc.) is nevertheless representative of the general case.

Serial Models

All of n *must finish.* Clearly, if comparisons are made in a serial fashion, then all n of them must be made with a total time nT. If the serial model is exhaustive, so this stopping rule is always followed, then the prediction is that for both positive decisions (the critical item is in the memory or display set) and negative decisions (the critical item is not in the set), one predicts that positive and negative RT functions as a function of n will be parallel (i.e., with the same slope) as often observed in memory-scanning studies.

One particular of n *must finish.* If serial searches are self-terminating, then positive RT functions would have half of the slope of negatives, since only half of the items must be searched on the average, whereas on negative trials all items must be searched.

Any one of n *must finish.* For serial self-terminating search, only the first item need be compared with the probe to constitute a match, so the time would be T, independent of n. Although no memory search paradigms seem to have used this procedure, the following two examples might be provided:

[1] For example, parallel unlimited capacity exponential models with independent comparison times predict negatively accelerated mean exhaustive reaction time (RT) curves as n increases. It is straightforward to show that any parallel unlimited capacity model with independent comparison times, irrespective of the distribution, predicts such negatively accelerated mean RT functions of n. Let $G(t)$ be the comparison time distribution for each item, for all values of n. Then the mean processing time for a given n can be expressed

$$E(t|n) = \int_0^\infty [1 - G^n(t)]dt$$

and the second order difference as

$$\Delta^2 E(t|n) = \int_0^\infty [2G^n(t) - G^{n+1}(t) - G^{n-1}(t)]dt,$$

which is readily seen to always be less than zero, thus proving negative acceleration.

(a) Suppose all memory search items are the same and the probe is positive (hence it matches all items); (b) suppose items are either digits or letters, and the subject's task is to decide whether the probe is a member of that class. In both tasks a single comparison provides the needed information for both negative and positive matches.

Unlimited Capacity Parallel Models

The parallel predictions are based on the assumption that comparison times are exponentially distributed and that all comparisons have the same rate parameter. Although restricting our attention to exponential distributions limits the generality of the parallel results, we can expect the qualitative form of the parallel predictions to be applicable to distributions of comparison times other than the exponential (see Footnote 1). To discuss the predictions for the parallel models, we first review some properties of exponential distributions.

1. For a single exponential distribution of comparison times t, $f(t) = ae^{-at}$, where $a > 0$, (a constant), the mean is $1/a$ (and corresponds to T), and the variance is $1/a^2$.

2. If a particular comparison x, with mean $1/a$, must be completed *and* if the exponential rate is independent of the number of potential comparisons, then the mean time for that comparison to be completed equals $1/a$ ($= T$) regardless of whether any of the other comparisons have finished. Thus, the prediction for Stopping Rule b for unlimited capacity parallel models is simply T. (Note that unlimited capacity in a parallel model means that the rate parameter is unaffected by the number of potential comparisons.)

3. When a number of such exponential distributions are sampled simultaneously (i.e., in a parallel system), we can take advantage of one property of exponential distributions to derive distributions of intercompletion times, namely, that exponential distributions have no memory. That is, given that by some critical time t_c, a particular comparison x, with mean $1/a$, has not been completed, the mean for that comparison time as measured from time t_c *remains* $1/a$. Thus, it is simple and will be fruitful to consider the ***intercompletion*** times (e.g., Townsend, 1974) among completed comparisons in making our predictions.

4. For Stopping Rule c, the time for the *first* of two comparisons to finish, whose rate parameters are a_1 and a_2, is $1/(a_1 + a_2)$. Stopping Rule c describes a horse race on an infinitely wide track, in which the time for the fastest horse is the crucial variable. As long as the running times have nonzero variance, increasing the number of horses (or comparisons) in the race leads to a decrease in the time of the fastest horse (or comparison). In particular, if the times have exponential distributions and the rate parameters are all equal ($1/a_i = 1/a$), the time for the first to finish is $1/(na)$, or since $1/a = T$, $(1/n)T$.

5. From 4, we can derive the predictions for Stopping Rule a in the case of unlimited capacity. We do this by considering intercompletion times. When all n comparisons must finish, the time for the first one to finish is $1/na$; the time for the second, $1/(n-1)a$; and the time for the mth, $1/(n-m+1)a$. Thus the time for all to finish may be found by summing the intercompletion times, or

$$1/na + 1(n-1)a + 1/(n-2)a \ldots + 1/a,$$

or

$$1/a(1/n + 1/(n-1) \times 1/n - 2) + \ldots + 1/1) \simeq 1/a \log n.$$

Hence, since $1/a = T$, Stopping Rule a prediction is $(\log n)T$.[2]

Limited Capacity Parallel Models

There are several reasons for considering limited capacity parallel models as reason-

[2] Actually the appropriate approximation is

$$\frac{1}{a}\sum_{i=1}^{n}\frac{1}{i} \simeq \frac{1}{a}(\log_N n + .6),$$

where $\log_N$ is the natural logarithm. In the present instance we are mainly concerned with the basic *form* of the function (e.g., slope, etc.) and hence safely ignore the extra constant.

able candidates for search processes. First, the notion that attention might be distributed across a number of items, even though all of them are processed simultaneously, has a long history (see Neisser, 1963b).

Second, predictions of unlimited capacity parallel search models are often patently falsified by extant data. They predict that for memory scanning, positive responses either do not increase with n (for self-terminating scans) or increase with the log of n (for exhaustive scans). Negative responses are always predicted to increase with the log of n because search is always exhaustive. However, good evidence exists for the linearity, or near-linearity, of RT with set size for both positive and negative responses in the memory-scanning literature.

On the other hand, there is at least one situation in which evidence for unlimited capacity parallel search has been obtained, namely, visual search through sets of redundant targets. For a fixed search set, increasing the number of redundant targets decreases both error rate (Estes & Taylor, 1966) and RT (van der Heijden & Menckenberg, 1974), a result consistent with either serial or parallel visual search. However, increasing the number of redundant targets when the search set is concomitantly increased (and contains *only* targets) *also* decreases RT (van der Heijden, 1975), a result apparently compatible *only* with a parallel unlimited capacity process (or at least one that is less limited than the models we next consider).

However, for much of the extant data that show approximately linear increases in RT with increases either in memory set size or display set size, more reasonable parallel models would be of limited capacity in which a fixed capacity is divided across the items that must be processed. In lieu of any detailed information on the allocation of attention, it seems reasonable to assume that this fixed capacity is uniformly distributed across the possible comparisons. This means that the comparison rates are inversely related to the number of comparisons; for example, if $1/a$ is the time for a single comparison for a single item, then n/a is the mean time for a single comparison among n items. Thus, all of the predictions for parallel unlimited capacity comparisons in Table 1 simply get multiplied by n for the limited fixed capacity predictions.

These predictions are based on the assumption that the rate parameter remains constant throughout the series of comparisons, that is, the basic rate parameter is affected only by the total number of potential comparisons present at the beginning of the comparison process, and completion of one comparison does not thereby free attention so that it can be reallocated to the remaining items. This fixed capacity assumption implies that the item comparison times are stochastically independent.

Another alternative is to assume that as comparisons are completed, attention may be reallocated to the remaining items, and thus, as fewer and fewer items remain to be processed, the rate of processing for any one speeds up in proportion to the number remaining. This parallel model we term the *reallocatable capacity model.* It is mathematically equivalent to (and hence experimentally indistinguishable from) an exponential serial model with equal preferences on all processing orders (Townsend, 1972, 1974), and it has been proposed by Atkinson et al. (1969). However, it will be useful to derive its predictions from a parallel point of view.

Consider Stopping Rule a in which all n must finish. The time to finish the first comparison is $n \times 1/na$. That is, the unlimited capacity mean latency for the first comparison, $1/na$, is multiplied by the total number of comparisons simultaneously processed, n; for the second, it is $(n - 1) \times 1/(n - 1)a$; and so on. So the total time for all to finish is

$$n/na + (n - 1)/(n - 1)a + \dots + 1/a,$$

or

$$n(1/a) = nT$$

Another way of seeing the logic behind this result is to attach a total capacity of a to the system. The first term is constructed by noting that this capacity a is divided equally among each of the n items; thus, each individual *rate* is a/n, and the total rate for the first intercompletion time is $n \times a/n = a$. The first intercompletion time itself is, of course, just the reciprocal of the latter quantity, $1/a$. The analysis proceeds likewise for each succeeding intercompletion time (e.g., on the second there are $n - 1$ items sharing the capacity denoted by a). Thus we see that by the reallocation of attention rule, the time for any particular comparison in a series to finish is simply $1/a$ for an exhaustive system.

The self-terminating rule (Stopping Rule b) predicts that the time between any two completion times, regardless of order of finish, is $1/a = T$. This constant intercompletion time is multiplied by the average number of comparisons that have to be made before the "critical" item is found. But as in the serial case, the processes must go halfway through the list on the average, resulting again in the time $(n + 1)/2 \times T$. For Stopping Rule c, the time for the first item to finish is simply T.

As Table 1 illustrates, the mean RT predictions for the parallel reallocatable attention model are identical to those from the serial model. In addition, these models are actually equivalent in their distribution on finishing times and thus cannot be empirically tested against one another.

Experimental Tests

To provide rigorous experimental tests of the various classes of models, we selected exemplary stimulus–response configurations from among the prevailing experimental paradigms. Because we felt it desirable to test models across, rather than within, specific paradigms, we constructed conditions in which only one or two items were in short-term memory and one or two in a visual display. By varying the decision rules, we varied the number of comparisons needed to reach a decision.

The first experiment used matrix patterns of regular design as stimuli, whereas the second experiment used letters. Both experiments used the same eight comparison tasks, although they differed in details

Table 2
The Eight Experimental Conditions, Where the Top Stimulus is Presented First and the Bottom is Presented Second

A		B	
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	SAME	$\begin{bmatrix} 1 \\ 2\ 1 \end{bmatrix}$	RIGHT
$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	DIFF	$\begin{bmatrix} 1 \\ 1\ 2 \end{bmatrix}$	LEFT
C		D	
$\begin{bmatrix} 2\ 1 \\ \ \ 1 \end{bmatrix}$	RIGHT	$\begin{bmatrix} 2\ 1 \\ 3\ 1 \end{bmatrix}$	RIGHT
$\begin{bmatrix} 1\ 2 \\ 1\ \ \end{bmatrix}$	LEFT	$\begin{bmatrix} 1\ 2 \\ 1\ 3 \end{bmatrix}$	LEFT
E		F	
$\begin{bmatrix} 1 \\ 1\ 2 \end{bmatrix}$	SAME	$\begin{bmatrix} 1\ 2 \\ 1 \end{bmatrix}$	SAME
$\begin{bmatrix} 1 \\ 2\ 1 \end{bmatrix}$	SAME	$\begin{bmatrix} 2\ 1 \\ 1 \end{bmatrix}$	SAME
$\begin{bmatrix} 1 \\ 2\ 3 \end{bmatrix}$	DIFF	$\begin{bmatrix} 2\ 3 \\ 1 \end{bmatrix}$	DIFF
G		H	
$\begin{bmatrix} 1\ 2 \\ 1\ 2 \end{bmatrix}$	SAME	$\begin{bmatrix} 1\ 2 \\ 1\ 3 \end{bmatrix}$	SAME
$\begin{bmatrix} 1\ 2 \\ 2\ 1 \end{bmatrix}$	SAME	$\begin{bmatrix} 1\ 2 \\ 3\ 1 \end{bmatrix}$	SAME
$\begin{bmatrix} 1\ 2 \\ 1\ 3 \end{bmatrix}$	DIFF	$\begin{bmatrix} 1\ 2 \\ 3\ 4 \end{bmatrix}$	DIFF
$\begin{bmatrix} 1\ 2 \\ 3\ 1 \end{bmatrix}$	DIFF		

Note. The convention here is to denote a single matching stimulus as "1" and two matching stimuli as "1" and "2," although in the experimental procedure, the particular matching stimuli were counterbalanced across the five possible stimuli. DIFF = different.

of stimulus presentation. The results of these experiments will be used for two purposes: first, to test whether subjects behave in a self-terminating or exhaustive manner at a qualitative level, and second, to illustrate in some detail how quantitative predictions for serial and parallel models with exponentially distributed comparison times are derived.

Table 2 presents the set of eight tasks used in the two experiments. The numbers refer to identical or different stimuli, and the correct response is shown to the right of each pair. The top stimulus in each group is presented first, and the bottom stimulus is presented shortly thereafter, remaining in view until the subject responds. Thus, some representation of the first stimulus resides in short-term memory, whereas some representation of the second stimulus resides in the perceptual store; so according to our previous terminology, all tasks are $n:m$, where both n and m take on values of either 1 or 2.

Condition A is a simple same–different task; Conditions B, C, and D are match-location tasks; Condition E is a visual scan task with two items in the visual display; Condition F is a memory scan task with two items in short-term memory; Condition G is a conjunctive same–different task; and Condition H is a disjunctive same–different task.

For Conditions B, C, and D, a match is always present and the subject's task is to indicate the location of the match, hence our designation of these conditions as *match-location* tasks. These conditions represent, for self-terminating models, Stopping Rule c in which the completion of any comparison is sufficient to make a decision. For all three conditions it is always the case that an item on the right or left matches the single item (for B and C) or one of the two items (for D), although B and C differ by whether the single item was presented first or second. Thus, finding a mismatch between corresponding positions is as informative as finding a match. Non-super-capacity exhaustive models all predict that as the number of possible comparisons increases, RT will increase; so comparisons of Conditions B, C, and D with A are diagnostic in choosing between self-terminating and reasonable exhaustive models. Furthermore, an unlimited capacity self-terminating parallel model predicts that as the number of possible comparisons increases for Stopping Rule c, RT will decrease; so, again, comparisons between B, C, D, and A are diagnostic in deciding between limited and unlimited capacity parallel models. Finally, both self-terminating serial and self-terminating fixed capacity parallel models predict no difference between Conditions B, C, D, and A.

Conditions E and F represent, respectively, visual search and memory search conditions for which $n = 2$. The two conditions are logically identical, with the exception that Condition E has one item in memory and two in the display (1:2), whereas Condition F has two items in memory and one in the display (2:1). So on positive, or *same*, trials self-terminating models predict that Stopping Rule b will be used (i.e., the subject can terminate only when he has completed the positive match), whereas on *different* trials all models predict that Stopping Rule a will be used (i.e., all comparisons must be completed before responding). Because under a self-terminating model, both E and F require more comparisons than Conditions A–D, both serial and parallel fixed-capacity self-terminating models predict that these conditions should take longer than Conditions A–D. Furthermore, since same decisions can be made after an average of only one and one-half comparisons, whereas different decisions require two comparisons, same decisions should be faster than different decisions as long as same and different rates do not differ too much.

Conditions G and H represent, respectively, conjunctive same–different and disjunctive same–different tasks. On Condition G same trials, all models predict that Stopping Rule a will be used—All matches must be completed before a decision can be made. For different trials, on the other hand, self-terminating models predict that a modified version of Stopping Rule b will be used; as soon as two mismatches are completed—those between the odd item (i.e., 3) in the display and *both* items in the memory set—the decision is made.

On Condition H same trials, on the other hand (which are physically but not logically identical to Condition G different trials), a single match between an item in the display and an item in memory is sufficient (Stopping Rule b), whereas for H different trials, all mismatches must be completed (Stopping Rule a) before a decision can be made.

Two experiments were run to test the predictions of the various models. Both experiments included all eight conditions depicted in Table 2. However, Experiment 1 used visual patterns as stimuli (simple matrix patterns composed of black and white squares), whereas Experiment 2 used single letters as stimuli. Experiment 1 has already been reported in detail elsewhere (Snodgrass, 1972b); however, no attempt was made to fit a parallel model to those data at that time.

Method

Subjects

Five subjects served in each experiment; one in Experiment 1 and two in Experiment 2 were female. All subjects were right-handed and were paid for their participation.

Stimuli

In Experiment 1, stimuli were five matrix patterns of black-and-white squares, rated as simple in previous experiments and chosen to be highly discriminable from one another. In Experiment 2, stimuli were the five uppercase letters D, Q, R, T, and Z.

Apparatus

In Experiment 1, the subject was seated in a darkened room and viewed the patterns (projected as slides on a screen) through the one-way mirror opening into an adjacent room where the experimenter operated the projection and recording equipment. The slides were projected by three Kodak Carousel slide projectors. The middle projector presented the single stimulus, and the two flanking projectors were used for paired stimuli. A pair of stimuli subtended approximately 6° of visual angle in the horizontal direction.

The stimulus presentations were actuated manually, so the stimulus durations, interstimulus interval, and intertrial interval were only approximate. The first stimulus was exposed for approximately $2\frac{1}{2}$ sec; the interstimulus interval was approximately $2\frac{1}{2}$ sec, and the intertrial interval was approximately 6 sec. The second stimulus remained exposed until about 1 sec after the subject had responded.

The release of a slide in one of the projectors used for presenting the second stimulus actuated a microswitch that started a Hunter Klockounter. The subject's press of one of two standard telegraph keys stopped the clock, and RT was recorded to the nearest msec. A light on the experimenter's console and one on the desk at which the subject was seated indicated which key had been pressed.

In Experiment 2, stimuli were presented via a Scientific Prototype automatic three-channel tachistoscope (Model GB) equipped with a binocular zoom lens (Kalimar K 7012). The stimuli were photographed from letters mounted on white cards (Instantype L-1510) and were made into 35-mm black-and-white slides. Single letters appeared in the middle of the slide, and letter pairs were separated by $\frac{1}{8}$ in (.32 cm). The five single letters were uppercase D, Q, R, T, and Z, and letter pairs were all 20 possible ordered pairs of the five. When projected in the tachistoscope, a single letter subtended between 1.75° and 3.5° of visual angle horizontally and 3.5° vertically, and a pair subtended between 5.5° and 9° horizontally.

The stimulus durations, interstimulus interval, and intertrial interval were controlled automatically by three time-interval generators. The first stimulus was presented for 2 sec, followed by a $2\frac{1}{2}$ sec blank lighted field, followed by the second stimulus, which was exposed for 2 sec. A dark blank field followed the end of the second stimulus for 3 sec and served as the intertrial interval.

The onset of the second stimulus started an electronic counter (Montsanto Counter-Timer No. 101b), and a press of one of two response keys by the subject stopped the timer and displayed the RT to the nearest msec. A light on the experimenter's console displayed which response key the subject had pushed.

The subject was seated at a table and viewed the stimuli through the binocular zoom lens, which had rubber eye cups. Each eyepiece was focused independently by each subject before each session. The experimenter was seated behind the tachistoscope in the same room with the subject and started and stopped the stimulus presentations, recorded RTs, and informed the subject when he or she had made an error. Automatic changers in both fields advanced slide trays (Sawyers Rototray) containing the sequence of stimuli for each session.

Design and Procedure

Other than the procedural differences due to different apparatus outlined in the *Apparatus* section, both experiments used identical designs and procedures. Each subject in both experiments participated in 3 practice and 18 experimental sessions. A complete cycle of the eight experimental conditions was completed in three sessions. Thus each subject experienced each condition once during practice sessions and six times during experimental sessions.

The conditions were divided into three sets to minimize interference. The three conditions requiring location information—B, C, and D—were run in one session, A and G in a second, and E, F, and H in a third. The order in which the three sets of conditions were run and the order of conditions within a particular session were completely counterbalanced across the 18 experimental sessions and were the same for each subject. Because we wished to equalize

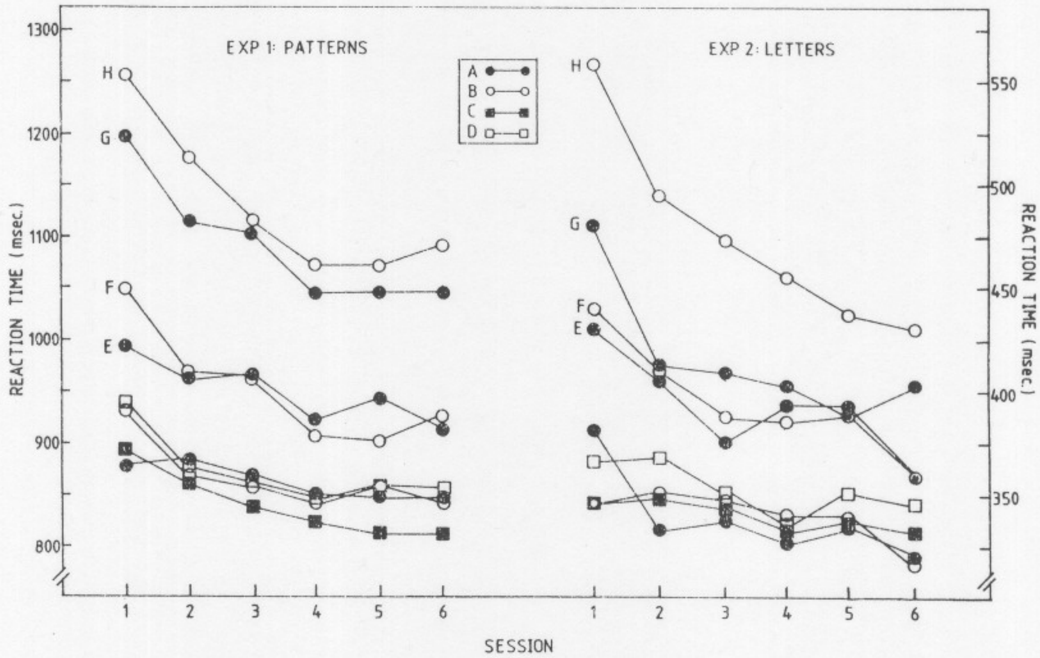

Figure 1. Mean correct reaction times by session for each of the two experiments. (A, B, C, D, E, F, G, H refer to conditions in both experiments.)

the number of possible trial types within a condition, the more complex conditions required more trials per session than the simpler conditions. Specifically, a particular session for conditions A, B, C, E, and F consisted of 40 trials, for D and H of 60 trials, and for G of 80 trials.

For each condition in each session, the number of SAME and DIFFERENT or RIGHT and LEFT trials was equal, all single and paired stimuli occurred equally often, and the spatial locations of matching stimuli were counterbalanced across locations. For Experiment 1 a single basic sequence for each condition was constructed subject to the above constraints and then was permuted in three ways to yield four distinct sequences. For Experiment 2 seven different random sequences for each condition were constructed by computer and were used for the seven presentations of each condition in a random order, which was generally the same for each subject.

A typical session lasted approximately 1 hr. Prior to running the experimental trials for each condition, 10 practice trials, selected randomly from the experimental sequence, were run to familiarize the subject with the condition. In addition, each subject was provided with schematic diagrams of all conditions to which he or she could refer during the experiment.

Subjects were paid $1.50 for participating in each session and, in addition, won money for fast responses and were penalized for errors. Because the experimental procedure was relatively complex, no counterbalancing of hand with responses was attempted. Instead, each subject used the apparently natural mapping of right hand for a RIGHT/SAME response and left hand for a LEFT/DIFFERENT response.

Results

Learning Effects

Figure 1 presents RTs for each condition plotted as a function of sessions for both experiments. The RTs are averages of both responses. Both experiments show similar patterns of results.

First, fairly large decreases in RT took place across sessions, and the decrease was larger for the more complex conditions such as G and H than for the simpler conditions such as A, B, C, and D. Whereas RTs for all conditions appear to be asymptotic by the fourth session for Experiment 1, RTs for Condition H in Experiment 2, which is clearly the most difficult condition in this experiment, show steady and regular decreases, which appear not to be asymptotic even by the sixth session.

Second, in both experiments Conditions A, B, C, and D are virtually indistinguishable from one another and show little decrease with sessions. Similarly, for both experiments Conditions E and F are virtually indistinguishable, although there is some decrease in RTs with sessions.

Finally, the main difference in RTs between the two experiments, aside from the lack of an asymptote for Condition H in Experiment 2, is that Condition G appears to have been much more difficult in Experiment 1 than it was in Experiment 2, in which the RTs for Condition G approach the RTs for Conditions E and F.

It seemed useful to fit models only to asymptotic data. The last three sessions from Experiment 1 are clearly asymptotic, so those three sessions were combined and the resulting data was used in preliminary analyses.

To determine whether the last three sessions for Experiment 2 were asymptotic, we performed a three-way repeated-measures analysis of variance on the data for the last three sessions, with experimental condition, response (where RIGHT and SAME are considered as one class and LEFT and DIFFERENT as the second), and session as the factors. Both the main effect of condition and of response were highly significant, whereas the main effect of session was not. For condition, $F(7, 28) = 23.34$, $p < .01$. For response, $F(1, 4) = 74.82$, $p < .01$. The Condition × Response interaction was significant, $F(7, 28) = 9.55$, $p < .01$, but none of the other interactions, Condition × Session, Response × Session, or Condition × Response × Session, was significant. Thus the data averaged across subjects for the last three sessions of Experiment 2 may be considered at least statistically asymptotic, and these will be used in preliminary analyses of the various models.

Asymptotic results. Table 3 presents mean RTs and error rates for the last three sessions of both experiments. In general, error rates are acceptably low, although errors tend to increase from simpler to more complex conditions (as do RTs). Although RTs for the letter study are considerably lower than those for the pattern study, due largely to the different apparatus used, there is a remarkable similarity between

Table 3
Correct Reaction Times (RTs) and Error Rates for the Last Three Sessions of Experiments 1 and 2

	Experiment 1 (patterns)		Experiment 2 (letters)	
Condition	*M* RT	% errors	*M* RT	% errors
A				
SAME	833	1.33	299	1.00
DIFF	869	1.00	356	3.33
B				
RIGHT	832	.67	325	1.00
LEFT	869	1.00	341	1.67
C				
RIGHT	813	.67	320	1.67
LEFT	832	2.33	347	1.33
D				
RIGHT	835	1.11	339	.22
LEFT	874	2.44	353	2.44
E				
SAME	917	5.00	358	2.67
DIFF	942	1.00	408	3.00
F				
SAME	886	6.67	351	4.00
DIFF	943	2.33	408	3.67
G				
SAME	1,035	6.33	368	2.33
DIFF	1,043	6.67	429	2.17
H				
SAME	1,025	6.89	401	5.78
DIFF	1,133	3.33	485	5.11

Note. DIFF = different.

the RTs obtained in the two studies. The Pearson product-moment correlation between the 16 pairs of RTs from the two studies is .89 ($p < .001$); the correlation is higher across the DIFFERENT or LEFT responses than across the SAME or RIGHT responses ($r = .97$ and .87, respectively).

In general, RIGHT or SAME RTs are faster than their corresponding LEFT or DIFFERENT RTs. This does not seem to be completely attributable to a difference between the overt physical responses, however, since the magnitude of the difference tends to be greater for SAME–DIFFERENT than for RIGHT–LEFT responses. For Experiment 1, the SAME–DIFFERENT difference is 47 msec, compared to 32 msec for the RIGHT–LEFT difference; the corresponding differences for Experiment 2 are 62 and 19 msec, respectively.

Stages in Fitting the Models to the Asymptotic Data Base

As we pointed out in the introduction, there are large classes of serial and parallel models possible for any given data base. The approach taken here is to fit models to the present data in several stages. We first used the qualitative pattern of empirical results to determine which broad classes of models could not be rejected and then compared these models according to their quantitative fits.

As noted previously, comparisons of RTs from the first four conditions in the experimental paradigm can distinguish self-terminating from exhaustive models, and unlimited capacity from limited capacity parallel models. To summarize, the RT patterns predicted by four broad classes of models for Conditions A–D are as follows: serial exhaustive, $A < B = C = D$; serial self-terminating, $A = B = C = D$; unlimited capacity parallel self-terminating, $A > B = C = D$; limited capacity parallel self-terminating, $A = B = C = D$.

The lack of an increase in RT as the number of possible comparisons increases (from A to B, C, and D) leads us to a self-terminating model, which in the parallel case must be of limited capacity, since an unlimited capacity model predicts a decrease in RT as the number of critical comparisons increases. Accordingly, we restrict our quantitative predictions to self-terminating limited capacity models.

Spatial position effects. Before developing the mathematical apparatus necessary to test quantitative differences between the appropriate serial and parallel models, we first examine the fine structure of the results to determine what they say about the strategies adopted by subjects to search for matches or mismatches. One of the advantages of using conditions for which very few potential matching locations exist is the possibility of analyzing these strategies in detail. The implication of spatial position effects for a serial model is that subjects have a preferred order of search and for a parallel model that attention is not allocated uniformly across the possible matching positions.

The conditions for which spatial position effects can occur are the more complex Con-

Table 4
Mean Correct Reaction Times for the Various Matching Locations Indicated in the Diagram for Conditions E, F, G, and H, Based on the Last Three Sessions Only

Condition	Configuration	Experiment 1	Experiment 2
E			
SAME	$\begin{bmatrix} & 1 & \\ 1 & & 2 \end{bmatrix}$	956	371
SAME	$\begin{bmatrix} & 1 & \\ 2 & & 1 \end{bmatrix}$	882	345
F			
SAME	$\begin{bmatrix} 1 & & 2 \\ & 1 & \end{bmatrix}$	905	362
SAME	$\begin{bmatrix} 2 & & 1 \\ & 1 & \end{bmatrix}$	867	340
G			
SAME	$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$	980	357
SAME	$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$	1,109	378
DIFF	$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$	1,052	431
DIFF	$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$	1,060	435
DIFF	$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$	1,059	422
DIFF	$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$	989	424
H			
SAME	$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$	1,054	396
SAME	$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$	962	385
SAME	$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$	1,038	409
SAME	$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$	1,065	410

Note. DIFF = different.

ditions E, F, G, and H. Table 4 presents mean RTs for the last three sessions of both experiments for various matching locations of Conditions E, F, G, and H. With some exceptions (G DIFF and H SAME trials), the two sets of data show similar spatial order effects (the Pearson product-moment correlation between the 14 pairs of RTs is .74, $p < .01$). For Conditions E and F, SAME RTs are faster when the matching stimulus is on the right. For Condition G, SAME RTs are faster when matching stimuli occupy corresponding spatial positions. The high correlation between the two sets of data indicates that the basic pattern of results is robust across stimuli and subjects and suggests that the same basic processes are involved in the two experiments.

Implications for a serial model. If a serial self-terminating model is assumed, these results suggest that (a) the comparison process begins with the right item in both memory and perceptual stores; (b) corresponding spatial positions are compared first, followed by diagonal positions if necessary; and (c) the memory store is searched for every display item. Conclusion a accounts for results of E and F. Conclusion b accounts for results of G SAME, since matches for corresponding positions $\begin{bmatrix}1 & 2\\1 & 2\end{bmatrix}$ will be found more quickly than for diagonal positions $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$. Conclusions b and c account for the fact that H SAME trials of type $\begin{bmatrix}2 & 1\\3 & 1\end{bmatrix}$ are faster than the other types. However, without some assumption about the interaction of items, these assumptions predict that for G DIFF, the configuration $\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$ should be equal to $\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$, since the subject will begin (or tend to begin) with Item 3, search through the two items in memory, and fail to find a match on either, so he or she can make his or her DIFFERENT response on that basis. Although these assumptions will not account for all the data, they provide some framework for building a model.

Implications for a parallel model. If a parallel self-terminating model is assumed, these results suggest that rates for comparing memory with display items depend on spatial location and are faster for right pairs than left pairs of items and slowest for items on the diagonal. These conclusions correspond to Conclusions a and b in the serial model. There appears to be no parallel conclusion analogous to c, that the memory store is searched first.

General implications for both classes of models. Predictions for both classes of models assume that there is some residual latency, t_R and t_L for right-hand and left-hand responses, respectively, that represents the sum of stimulus input, response organization, and response output time. We assume that the time to perform the relevant comparisons is added to this residual time without interacting with any of its components. For both models, the following assumptions are explicitly made in deriving the prediction equations. First, we assume that the subject keeps track of both the location of matching (or mismatching) items and whether they were matches (same items) or mismatches (different items). Second, we assume that the subject can control his or her attention so that classified items can be eliminated from further consideration. We illustrate how these assumptions are applied by deriving the serial equation for the G SAME trials of the form $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$ for a simplified serial model that assumes the subject always begins comparisons with items on the right (ultimately, the probability of beginning with the right position will become a parameter in the model). Recall that to come to a same decision, two matches must be found. Because items in spatially identical positions are compared first, the first comparison is the different comparison 2:1, and the second is the same comparison 1:1. At this point the subject eliminates items in the 1:1 comparison from further consideration and remembers that one match has been made. The only remaining comparison is 2:2, leading to the second match and a decision of same. In the next sections we present in detail the assumptions of the two models and derive their predictions in detail. Because spatial position effects are large and consistent for the more complex conditions, we attempt to predict latencies for each distinct configuration in these conditions, rather than merely predicting either overall

Table 5
Predicted Mean Latencies for the Serial Self-Terminating Model

Condition	Configuration	Predicted RT
A		
SAME	$\begin{bmatrix}1\\1\end{bmatrix}$	(1) $t_R + 1/s$
DIFF	$\begin{bmatrix}1\\2\end{bmatrix}$	(2) $t_L + 1/d$
B, C, D		
RIGHT	$\begin{bmatrix} & 1 & \\ 2 & & 1\end{bmatrix} \begin{bmatrix}2 & & 1\\ & 1 & \end{bmatrix} \begin{bmatrix}2 & 1\\3 & 1\end{bmatrix}$	(3) $t_R + P(1/d) + (1 - P)1/s$
LEFT	$\begin{bmatrix} & 1 & \\ 1 & & 2\end{bmatrix} \begin{bmatrix}1 & & 2\\ & 1 & \end{bmatrix} \begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$	(4) $t_L + P(1/s) + (1 - P)1/d$
E, F		
SAME	$\begin{bmatrix} & 1 & \\ 1 & & 2\end{bmatrix} \begin{bmatrix}1 & & 2\\ & 1 & \end{bmatrix}$	(5) $t_R + P(1/s) + (1 - P)(1/s + 1/d)$
SAME	$\begin{bmatrix} & 1 & \\ 2 & & 1\end{bmatrix} \begin{bmatrix}2 & & 1\\ & 1 & \end{bmatrix}$	(6) $t_R + P(1/s + 1/d) + (1 - P)1/s$
DIFF	$\begin{bmatrix} & 1 & \\ 2 & & 3\end{bmatrix} \begin{bmatrix}2 & & 3\\ & 1 & \end{bmatrix}$	(7) $t_L + 2/d$
G		
SAME	$\begin{bmatrix}1 & 2\\1 & 2\end{bmatrix}$	(8) $t_R + 2/s$
SAME	$\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$	(9) $t_R + 2/s + 1/d$
DIFF	$\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$	(10) $t_L + P(1/s + 1/d) + (1 - P)2/d$
DIFF	$\begin{bmatrix}2 & 1\\3 & 1\end{bmatrix}$	(11) $t_L + P(2/d) + (1 - P)(1/s + 1/d)$
DIFF	$\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$	(12) $t_L + P(2/d) + (1 - P)(1/s + 2/d)$
DIFF	$\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$	(13) $t_L + P(1/s + 2/d) + (1 - P)2/d$
H		
SAME	$\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$	(14) $t_R + P(1/s) + (1 - P)(1/s + 2/d)$
SAME	$\begin{bmatrix}2 & 1\\3 & 1\end{bmatrix}$	(15) $t_R + P(1/s + 2/d) + (1 - P)(1/s)$
SAME	$\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$	(16) $t_R + P(1/s + 3/d) + (1 - P)(1/s + 1/d)$
SAME	$\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$	(17) $t_R + P(1/s + 1/d) + (1 - P)(1/s + 3/d)$
DIFF	$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$	(18) $t_L + 4/d$

Note. DIFF = different. t_R, t_L = residual latencies for right and left responses. $1/s$, $1/d$ = mean latencies for same and different comparisons. P = probability of starting comparisons on the left.

mean RTs or mean RTs for each response. Separating conditions by their spatial configurations increases the degrees of freedom in fitting the models, and also provides a more rigorous comparison of the two models.

Serial Self-Terminating Model

Assumptions

(a) Search is through all memory items for each display item until a match is made or the memory store is exhausted. (b) Comparisons are first made for corresponding spatial locations, then for diagonal locations. (c) A counter keeps track of the number and spatial locations of matches, which are eliminated from further searches.

There are five parameters: t_R = residual latency for right response; t_L = residual latency for left response; $1/s$ = mean exponential latency for same comparison; $1/d$ = mean exponential latency for different comparison; P = probability of starting comparisons on left. (Note that in the simplified model described above, we set $P = 0$.)

Defining the mean comparison latencies as $1/s$ and $1/d$ means that the exponential rate parameters are s for same and d for different comparisons. Although the serial equations could be simplified by defining the mean comparison times as, for example, S and D, the use of the present notation preserves a parallel structure between the serial and parallel predictions.

Additionally, in these equations we have not distinguished between the match and mismatch rates as a function of spatial position. A more complex version of the model is also tested, which separately estimates left, right, and diagonal match and mismatch rates.

Equations for mean latencies are derived for 18 conditions. Some conditions (B, C, and D; E and F) have identical predicted latencies, whereas other conditions, such as G SAME and DIFFERENT and H SAME, are separated into separate configurations requiring different numbers of comparisons because of search order effects. Table 5 presents the prediction equations for the serial self-terminating model.

Limited Fixed Capacity Parallel Self-Terminating Model

Assumptions

(a) Intercompletion times are exponentially distributed. (b) Exponential rate parameters remain constant regardless of the stage of processing and, hence, regardless of the number of elements remaining to be processed. Thus, capacity is not reallocatable during a trial, and processing of elements is stochastically independent. (b) The model is basically fixed or constant capacity. That is, the rate parameters depend on the total possible number n of comparisons (one for A, two for B, C, D, E, and F, and four for G and H), such that the basic rate parameter is divided by n. However, to simplify the notation, we let the basic rate parameters s_R, s_L, d_R, and d_L be those for $n = 2$. Hence those rate parameters are used for Conditions B, C, D, E, and F, whereas for Condition A, in which only a single comparison is made, the rate parameter is doubled by adding s_R and s_L for same comparisons and adding d_R and d_L for different comparisons.

There are six parameters: t_R, and t_L, defined as for the serial model; $1/s_R$ = mean exponential latency for same comparison on right; $1/s_L$ = mean exponential latency for same comparison on left; $1/d_R$ = mean exponential latency for different comparison on right; $1/d_L$ = mean exponential latency for different comparison on left.

Note that for the parallel model, we *do* distinguish processing rates for right and left positions. This distinction must be made because of the strong spatial position effects, and it corresponds to the serial model assumption of a preferred processing order. In addition, in a more complex version of the model that is also tested, rate parameters for diagonal comparisons are distinguished from right and left comparison rates.

Table 6 presents an abbreviated version of the prediction equations for the parallel model. In the equations for Conditions B–F, the spatial position distinctions between rates are retained. However, the prediction equations for Conditions G and H

Table 6
Predicted Mean Latencies for the Parallel Self-Terminating Fixed Capacity Model

Condition	Configuration	Predicted RT
A		
SAME	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	(1) $t_R + \frac{1}{s_A}$, where $s_A = s_R + s_L$
DIFF	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	(2) $t_L + \frac{1}{d_A}$, where $d_A = d_R + d_L$
B, C, D		
RIGHT	$\begin{bmatrix} \multicolumn{2}{c}{1} \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ \multicolumn{2}{c}{1} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$	(3) $t_R + \frac{1}{s_R + d_L}$
LEFT	$\begin{bmatrix} \multicolumn{2}{c}{1} \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ \multicolumn{2}{c}{1} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$	(4) $t_L + \frac{1}{s_L + d_R}$
E, F		
SAME	$\begin{bmatrix} \multicolumn{2}{c}{1} \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ \multicolumn{2}{c}{1} \end{bmatrix}$	(5) $t_R + \frac{1}{s_L}$
SAME	$\begin{bmatrix} \multicolumn{2}{c}{1} \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ \multicolumn{2}{c}{1} \end{bmatrix}$	(6) $t_R + \frac{1}{s_R}$
DIFF	$\begin{bmatrix} \multicolumn{2}{c}{1} \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ \multicolumn{2}{c}{1} \end{bmatrix}$	(7) $t_L + \frac{1}{d_R + d_L} + P_R\left(\frac{1}{d_L}\right) + P_L\left(\frac{1}{d_R}\right)$, where $P_R = \frac{d_R}{d_R + d_L}$; $P_L = \frac{d_L}{d_R + d_L}$
G		
SAME	$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$	(8), (9) $\begin{aligned} & t_R + P_1\left(\frac{1}{2s' + 2d'} + \frac{1}{s' + 2d'}\right) + P_2\left(\frac{1}{2s' + 2d'} + \frac{1}{s' + 2d'} + \frac{1}{s' + d'}\right) + P_3\left(\frac{1}{2s' + 2d'} + \frac{1}{s' + 2d'} + \frac{1}{s' + d'} + \frac{1}{s'}\right) \\ & + P_4\left(\frac{1}{2s' + 2d'} + \frac{1}{2s' + d'} + \frac{1}{s' + d'}\right) + P_5\left(\frac{1}{2s' + 2d'} + \frac{1}{2s' + d'} + \frac{1}{s' + d'} + \frac{1}{s'}\right) + P_6\left(\frac{1}{2s' + 2d'} + \frac{1}{2s' + d'} + \frac{1}{2s'} + \frac{1}{s'}\right) \end{aligned}$
DIFF	$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$	(10), (11), (12), (13) $\begin{aligned} & t_R + P_1\left(\frac{1}{3d' + s'} + \frac{1}{3d'}\right) + (P_2 + P_3 + P_4)\left(\frac{1}{3d' + s'} + \frac{1}{2d' + s'}\right) + P_5\left(\frac{1}{3d' + s'} + \frac{1}{3d'} + \frac{1}{2d'}\right) + P_6\left(\frac{1}{3d' + s'} + \frac{1}{2d' + s'} + \frac{1}{2d'}\right) \\ & + P_7\left(\frac{1}{3d' + s'} + \frac{1}{2d' + s'} + \frac{1}{d' + s'}\right) + P_8\left(\frac{1}{3d' + s'} + \frac{1}{3d'} + \frac{1}{2d'} + \frac{1}{d'}\right) + P_9\left(\frac{1}{3d' + s'} + \frac{1}{2d' + s'} + \frac{1}{2d'} + \frac{1}{d'}\right) \\ & + P_{10}\left(\frac{1}{3d' + s'} + \frac{1}{2d' + s'} + \frac{1}{d' + s'} + \frac{1}{d'}\right) \end{aligned}$

Table 6 (*continued*)

Condition	Configuration	Predicted RT
H		
SAME	$\begin{bmatrix}1&2\\1&3\end{bmatrix}\begin{bmatrix}2&1\\3&1\end{bmatrix}$	(14), $t_R + \frac{1}{s'}$
		(15),
	$\begin{bmatrix}1&2\\3&1\end{bmatrix}\begin{bmatrix}2&1\\1&3\end{bmatrix}$	(16),
		(17)
DIFF	$\begin{bmatrix}1&2\\3&4\end{bmatrix}$	(18) $t_L + \frac{1}{4d'} + \frac{1}{3d'} + \frac{1}{2d'} + \frac{1}{d'}$

Note. DIFF = different. t_R, t_L = residual latencies for right and left responses. $1/s_R$, $1/s_L$ = mean latencies for same comparisons on right and left. $1/d_R$, $1/d_L$ = mean latencies for different comparisons on right and left. $1/s' = 4/(s_L + s_R)$; $1/d' = 4/(d_L + d_R)$.

are so complex that the spatial position distinctions are dropped in the equations as presented, although they were retained in fitting the model.

First, note that the prediction equations for A are similar to those for B, C, and D. Recall that for a fixed capacity parallel model, we halve the processing rate (and hence double the comparison time) each time the number of comparisons is doubled (as in going from A to B, C, or D). Thus, the comparison rate for A, s_A and d_A equals the sum of the left and right comparison rates for Conditions B, C, and D. Since the latter three conditions can logically terminate with one comparison, the comparison time is a "race" between same and different comparisons, with the winning comparison announcing a decision regardless of which comparison it is.

For E and F SAME, the only comparison of interest is the same comparison; hence we can ignore the finishing time of the different comparison in computing latency. For E and F DIFF, on the other hand, both comparisons must finish before a decision can be made. The latency with which the first comparison finishes, regardless of what it is, is $1/(d_R + d_L)$. The latency of the remaining comparison, $1/d_R$ or $1/d_L$, must be added to the latency of the first finishing. The probability that the right comparison finishes first, P_R, is $d_R/(d_R + d_L)$. The probability that the left comparison finishes first, P_L, is $d_L/(d_R + d_L)$. Thus the total latency is given by Equation 7 in Table 6.

For Conditions G and H, four comparisons are possible, so by the rule for fixed capacity models, we halve each of the rate parameters. The new rate parameters are denoted s'_R, s'_L, d'_R, and d'_L. Diagonal comparison rates are equal to the average of the right and left rates. For example,

$$s'_D = \frac{s'_L + s'_R}{2} \quad \text{and} \quad d'_D = \frac{d'_L + d'_R}{2}.$$

Both G SAME and G DIFF conditions are logically self-terminating, therefore their prediction equations become complex because each distinguishable processing order must be represented in the equations.

For G SAME decisions four comparisons, of which two are same and two are different, are possible. The subject is able to respond as soon as the two same comparisons have finished. There are six distinguishable finishing orders of two same and two different comparisons that contain the necessary and sufficient pair of same comparisons. Although the particular same or different comparisons that finish in a particular ordinal position are relevant, we ignore the spatial position distinctions for clarity and define s' as the rate for either same comparison and d' as the rate for either different comparison.

We remind the reader, again, that distinguishable orders of comparisons must be separately evaluated because the intercompletion times depend on the number of comparisons still to be completed. It should be emphasized that this dependence on number of remaining comparisons follows from the logical structure of the task and does not imply any change in the individual comparison rates as processing continues. As pointed out above, the latter are fixed.

In Equations 8 and 9 in Table 6 (which are identical because spatial position differences are ignored), the probabilities refer to the probabilities of particular finishing orders; P_1 is the probability that both s comparisons finish first, P_2 the probability of a finishing order sds, . . . and P_6 the probability of the finishing order ddss. By observing which term is omitted in each succeeding term in the equations and by keeping in mind the fact that two same comparisons are necessary, the complete equation can be constructed.

For example, the equations for P_1 and P_6 are as follows:

$$P_1 = \frac{2s'}{2s' + 2d'} \times \frac{s'}{s' + 2d'};$$

$$P_6 = \frac{2d'}{2s' + 2d'} \times \frac{d'}{2s' + d'} \times \frac{2s'}{2s'} \times \frac{s'}{s'}.$$

The first term in P_1 is the probability that either s comparison finishes first, and the second term is the probability that the remaining s comparison finishes next. The first two terms in P_6 refer to the corresponding probabilities for d comparisons, and the remaining two terms (both equal to 1.0 in this simplified version) refer to the probabilities that the remaining s comparisons finish.

For G DIFF trials four comparisons, of which three are different and one is same, are possible. However, in contrast to the case for G SAME trials, there are a variety of combinations of *particular* comparisons that can terminate the search and lead to a correct decision. To illustrate, we consider the particular case of $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. An observer can respond DIFF after completion of any of the following three sets of two comparisons (the first item is in memory and the second in the display): a same 1:1 and the different 2:3; the different 2:3 and the different 1:3; the different 2:3 and the different 2:1. It is important to note that it is not sufficient to complete the same comparison and *any* other different comparison; for example, the pair 1:1 and 2:1 will not yield sufficient information for a different decision. Similarly, any two different comparisons are not sufficient, as in the pair 2:1 and 1:3. Thus, although there are three possible different comparisons, one of them, which we will denote d_1, is crucial and must be completed along with either the same comparison or any other different comparison.

In Equations 10–13 in Table 6 (which are shown as identical equations because the differences in spatial processing rates have not been included there), P_1 gives the probability that the first two comparisons finished are sd_1, P_2 that the order is d_1s, and P_3 and P_4 that two differents finish first, including the critical one. P_5–P_7 denote the probabilities that d_1 finishes in third position, and P_8–P_{10}, the probabilities that d_1 finishes in fourth position. By examining the predicted intercompletion times and noting which comparison rate has been deleted as comparisons are completed, it is possible to infer the particular order postulated. For example, P_8 refers to the probability of the two orders $sd_2d_3d_1$ or $sd_3d_2d_1$, given by

$$P_8 = \frac{s'}{3d' + s'} \times \frac{2d'}{3d'} \times \frac{d'}{2d'} \times \frac{d'}{d'}$$

For H SAME trials the only comparison leading to a correct decision is the single same comparison; all of the different comparisons are irrelevant. Since the decision is self-terminating on the same comparison, the additional comparison time is simply $1/s'$. For H DIFF trials all four different comparisons must finish before the subject can make his or her decision. Hence this is a logically exhaustive task, and the prediction is as shown in Table 6.

Tests of the Models

The two classes of models were fit to individual subjects' data. To identify asymptotic data for individual subjects, mean RTs across sessions were plotted separately for each subject, and the asymptote was determined for each by inspection. The data that were fit by each model were the 18 RTs

Table 7
Serial and Parallel Self-Terminating Parameter Estimates for Experiment 1 (Patterns)

Subject	No. sessions	Parameters											
		t_R	t_L	$1/s$	$1/s_R$	$1/s_L$	$1/s_D$	$1/d$	$1/d_R$	$1/d_L$	$1/d_D$	P	χ^2
							Serial						
1	3	559	733	286				107				.27	34[a]
		598	694		348	89	250		296	0	89	.57	26[b]
2	3	679	732	148				106				.31	37[a]
		590	620		365	40	97		333	35	78	.66	21[b]
3	4	713	777	182				115				.21	37[a]
		615	693		183	310	146		121	221	100	.16	19[b]
4	2	660	717	170				117				.34	14[a]*
		625	681		203	204	142		152	143	104	.39	8[b]*
5	2	690	786	160				59				.26	29[a]
		616	679		93	275	104		25	236	42	.21	16[b]
							Parallel						
1	3	754	817		106	190			78	106			64[c]
		771	825		76	132	72		95	167	95		49[d]
2	3	774	800		74	127			86	85			49[c]
		774	797		73	124	95		92	104	75		46[d]
3	4	841	841		83	127			95	120			70[c]
		843	839		80	123	102		123	108	101		69[d]
4	2	773	793		77	129			86	112			17[c]*
		778	791		67	118	95		115	109	92		15[d]*
5	2	804	827		60	86			58	55			51[c]
		804	828		61	87	56		54	72	58		51[d]

Note. L = left, R = right, D = diagonal, t = residual response time, s = same comparison rate, d = different comparison rate, P = probability of starting on left. The first row lists parameter estimates for the 5-parameter version (serial) or 6-parameter version (parallel), and the second row lists parameter estimates for the 9-parameter version (serial) or 8-parameter version (parallel).
[a] $df = 13$. [b] $df = 9$. [c] $df = 12$. [d] $df = 10$. * $p > .05$.

whose prediction equations are shown in Tables 5 and 6 for those asymptotic sessions.

Two versions of each model were fit to these data; the serial self-terminating model could have either five or nine parameters, and the parallel limited fixed capacity self-terminating model could have either six or eight parameters. The different number of parameters was determined by whether same and different comparison rates were distinguished by spatial position for the serial model and whether diagonal comparison rates were estimated separately for the parallel model.

The function minimizing subroutine STEPIT (Chandler, 1959) was used to find the best-fitting parameter values. The function that was minimized was the chi-square, defined as

$$\chi^2 = \sum \frac{(M - \mu)^2}{\sigma^2/N},$$

where M is the observed mean RT; μ is the theoretical mean predicted from the model; σ^2 is the theoretical variance; and N is the number of observations on which the observed mean is based.

The theoretical variance was calculated

Table 8
Serial and Parallel Self-Terminating Parameter Estimates for Experiment 2 (Letters)

Subject	No. sessions	t_R	t_L	$1/s$	$1/s_R$	$1/s_L$	$1/s_D$	$1/d$	$1/d_R$	$1/d_L$	$1/d_D$	P	χ^2
		Parameters											
							Serial						
1	4	206	365	152				40				.13	133[a]
		206	255		17	187	26		45	195	16	.12	39[b]
2	5	346	353	15				55				.55	91[a]
		258	277		0	156	25		39	156	32	.21	39[b]
3	1	232	244	29				41				.45	20[a]
		176	197		36	109	19		44	104	23	.25	6[b]*
4	5	259	267	42				57				.51	179[a]
		206	215		26	132	0		51	140	35	.26	29[b]
5	3	185	253	116				71				.35	35[a]
		100	177		97	236	84		51	193	52	.23	12[b]*
							Parallel						
1	4	347	387		24	53			38	47			277[c]
		343	383		31	59	44		58	40	35		267[d]
2	5	351	376		59	29			35	52			234[c]
		351	374		52	28	41		59	46	32		217[d]
3	1	251	265		26	30			32	40			30[c]
		247	261		32	36	41		49	25	20		19[d]
4	5	286	300		42	39			36	64			267[c]
		278	294		54	46	46		78	38	37		236[d]
5	3	256	297		68	77			60	72			60[c]
		251	294		76	85	64		77	64	61		53[d]

Note. L = left, R = right, D = diagonal, t = residual response time, s = same comparison rate, d = different comparison rate, P = probability of starting on left. The first row lists parameter estimates for the 5-parameter version (serial) or 6-parameter version (parallel), and the second row lists parameter estimates for the 9-parameter version (serial) or 8-parameter version (parallel).
[a] $df = 13$. [b] $df = 9$. [c] $df = 12$. [d] $df = 10$. * $p > .05$.

as follows. The theoretical mean RT is assumed to be the sum of a base time (t_R or t_L) plus the sum of the exponentially distributed comparison times. The distribution of the base times was assumed to follow a gamma distribution with 100 stages, so its standard deviation is estimated as 10% of the estimated mean base time. The variance of the comparison times is based on the assumption of exponentially distributed comparison times and is equal to $1/a^2$ for a single comparison. The variance of the total time is the sum of the two variances.

The advantage of defining the chi-square in this manner is that it defines the sum of squared z scores and, as such, follows a true chi-square distribution. In contrast, the chi-square statistic that uses the empirical variance in the denominator is the sum of squared t scores, and it is not clear how closely that chi-square statistic follows a true chi-square distribution. An additional advantage of defining chi-square in this manner is that it makes the serial parameter estimation procedure partially dependent on the assumption of exponential distributions. That is, in predicting mean latencies for serial models, no restriction was imposed by the exponential assumption, in contrast to the parallel predictions. The theoretical variance estimates, in contrast, *are* influenced by the exponential assumption for *both* serial and parallel models and, hence, will affect both the parameter estimates and the goodness-of-fit estimate.[3]

Tables 7 and 8 present the estimated parameters and chi-square values for each subject for Experiments 1 and 2, respectively. For the serial five-parameter model, we do not distinguish between same comparisons on the right and left, since the parameter P can account for some of the serial position effects that are observed. In contrast, the parallel six-parameter model does not separately estimate the diagonal comparison rates. Rather, the diagonal rate was taken to be the average of the right and left rates.

For both Experiment 1 and Experiment 2, the serial self-terminating model fits the data considerably better than the parallel fixed capacity self-terminating model. For both models, the residual latency for right or same judgments (t_R) is estimated to be less than for left or different judgments (t_L), with two minor exceptions for the parallel model for Experiment 1. The serial model for patterns consistently estimates same comparison times (1/s) as longer than different comparison times (1/d), whereas the parallel model for both experiments and the serial model for letters shows no consistent relationship between same and different comparison times.

As expected, the probability of starting the comparison on the left (P) for the serial model is generally estimated as less than .5, with two exceptions for each of the two experiments. Also as expected, the parallel model predicts comparison times on the right to be shorter than comparison times on the left, particularly for Experiment 1, in which spatial position effects were most apparent.

Somewhat surprisingly, the serial nine-parameter model shows even larger differences between right and left comparison times. The magnitude of this difference is correlated with the value of P: For Experiment 1, in which right comparison times are sometimes shorter and sometimes longer than left comparison times, the correlation between the difference between the right and left comparison times $(1/s_R - 1/s_L)$ and P is .98 for same comparisons and .95 for different comparisons. For Experiment 2, in which left comparison times are always estimated as longer than right comparison times, the correlation is .78 for sames and .72 for differents. This analysis suggests that the preferred side (the side on which a subject more often began processing) also evidenced a faster processing speed than the nonpreferred side, whether it was the right or left side. Put another way, the parameter P is unable to absorb all of the spatial position effects; comparison rate differences are also required.

The finding that same comparisons take longer than different comparisons seems to be compatible with some recent notions put forth by Krueger (1978), although in some contexts it is the same comparisons that are faster (e.g., see Bamber, 1969; Townsend & Roos, 1973; Taylor, 1976a).

The serial model showed substantial changes in parameter estimates from the five- to nine-parameter versions. When combined with the considerable improvement in fit, this suggests that the more complex model is capturing structure in the data unaccounted for by the simpler version. In contrast, the parallel model showed less alteration in either its parameter values or in its fits from the six- to eight-parameter versions. Thus, overall, the parallel model seems to be incapable of appropriately modeling the patterns of RT, whether in its simpler or more complex form.

Both models show the largest deviations between predicted and observed RTs for the more complex conditions (particularly G and H), although the particular conditions on which they fail differ. Larger deviations on G and H are in part a consequence of the fact that those conditions are broken

[3] We also carried out the model fitting by defining χ^2 the usual way, with the empirical variance in the denominator, and found no substantive changes either in the pattern of parameter estimates or in the conclusions about which model fit better.

down into different spatial configurations and, thus, are based on fewer observations than the simpler conditions. Because the chi-square is weighted by the number of observations in each condition, deviations between predicted and observed RTs are weighted more heavily for the simpler conditions than for the more complex, and hence the parameters are adjusted to fit the simpler conditions in preference to the more complex conditions.

Since the models do not make the same predictions about mean latencies, it is clear that the serial and parallel models compared here are identifiably different, primarily because of the introduction of spatial position effects. Although how they differ in predicting latencies is difficult to see at the level of the formulas, an idea of how differences in prediction can come about can be gained from examining the equations of Tables 5 and 6. For example, Equations 3 and 4 sum to equal Equation 1 + Equation 2 in the simpler serial model, but not in the parallel model. Such differences, on a more subtle level, occur in the more complex conditions.

For example, the parallel model shows the largest failures in accounting for fast RTs in the G SAME configuration $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$, compared to the slower RTs in the configuration $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. For the parallel model to successfully predict faster times for the former than for the latter configuration, it must estimate diagonal comparison times as longer than nondiagonal comparison times. Yet other conditions demand that the diagonal times be shorter, and they are usually estimated to be intermediate to or smaller than the right and left matching times (see Tables 7 and 8). In contrast, the serial model rather naturally accounts for this difference by its assumption that corresponding spatial positions are compared first. Nonetheless, both models underestimate the differences between the two types of G SAME trials, even though the serial model fares better.

In contrast, the serial model has difficulty predicting the pattern of G DIFF times. As shown in Table 4, the G DIFF configuration $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ is typically faster than the G DIFF configuration $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, yet the serial prediction equations (Table 5, Equation 10 versus Equation 13) predict the opposite ordering regardless of the parameter values.

Although the serial model fits better than the parallel model, it can still be rejected as a complete account of subjects' processing strategies. Of the 10 estimates obtained with the complex version, only 3 were not significant at the .05 level. On the other hand, as Estes (1975) among others has noted, a sufficiently powerful experiment would reject all models, since no model of human information processing could be expected to capture all of the rich complexity of human behavior. The power of a statistical test increases with the number of observations, and it is largely up to scientific intuition as to the evaluation of a fit relative to the sample size. It is probably fair to say that the present experiments provided a reasonable challenge to mathematical models in terms of power and diversity of experimental conditions. One reasonable way to evaluate a model of human information processing is to compare it with a plausible alternative model whose underlying assumptions differ, as we have done here.

Discussion

In the past, mathematical serial models have often been confined to those assuming a fixed processing order and invariant processing times on the various elements. Mathematical parallel models of any variety have been rare. One of the purposes of the present article has been to explicate some of the mathematical structure of parallel and serial models that can be employed to represent various psychological notions and to show how parallel and serial models can be developed in a natural way for a given experimental context. Within this approach some nonparametric predictions made by fairly large classes of parallel and serial models were derived, with special attention to comparisons between self-terminating versus exhaustive processing rules and the limited versus unlimited capacity issue. Two experiments strongly pointed to self-terminating processing, whether serial or parallel, and then supported a plausible serial model against a plausible parallel model.

We have not, of course, tested all parallel against all serial models. Such a paradigm is not available, although the parallel–serial testing paradigm (PST) is capable of separating reasonably large classes of such models (Townsend, Note 1; Townsend & Snodgrass, Note 2).[4] The present models are distinctive in several respects and are, as the present findings indicate, experimentally discriminable. One important difference springs from the independence (and hence, nonreallocatability) of processing in the parallel model. This nonreallocatable property implies that the intercompletion times tend to lengthen as processing progresses, rather than staying the same overall as in standard serial and parallel models with complete reallocation. That is, as the number of potential comparisons decreases, the average intercompletion time increases as a consequence of the exponential process. Thus, later stages add more time to the RT than do the earlier stages. Another distinction is associated with the assumption that different comparisons consume a different amount of time, on the average, than do same comparisons (Townsend, 1976b, pp. 34–41). Finally, the large number of experimental conditions used here helps to provide a more rigorous test to any model.

Why, when parallel and serial models can give such similar accounts of data, do we think it important to distinguish them? First, because our theoretical understanding of underlying processes might then be better advanced, and second, because there are certain data in the literature that might be better understood if one model were to be preferred.

To give a single but important example, Shiffrin and Schneider (1977) and Schneider and Shiffrin (1977) have recently proposed a dichotomy of processing strategies in which a slower controlled processing is characterized by a serial self-terminating search and a faster automatic processing is characterized by a parallel unlimited capacity search. Which strategy is adopted by a subject is thought to be dependent on such factors as practice and constant versus variable mapping between stimuli and responses. However, such a switch from a serial to a parallel processing strategy implies a discrete, qualitative change in processing, rather than a quantitative change more typically attributed to effects of practice. It may be more in keeping with classical views of changes in skill levels to propose that controlled processing is characterized by a parallel but limited capacity search process, whereas the automatic mode reflects an increase in capacity of the system toward an unlimited capacity parallel system. This second view would hold that the ultimate nature of the processing does not change, only its quantitative parameters.

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[4] It has recently become clear that problems associated with parallel–serial model equivalence extend to more general domains than those simply based on exponential intercompletion times (Townsend, 1976a; Vorberg, Note 3). However, the two main distinctive aspects of the present models, parallel nonreallocatability and different rates for same and different comparisons, have immediate analogies in the general case that effectively prevent parallel-serial equivalence (Townsend & Ashby, Note 4). Townsend (1976a) and Vorberg (Note 3) do not consider these distinguishing aspects.

Reference Notes

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Received October 6, 1978 ■